

1. If $y = \frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x}$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is
- 0
 - 1
 - 1
 - 2
2. The value of $\cot\left(\frac{1}{2}(3\pi - \tan^{-1}(\frac{1}{3}))\right)$ equals
- $(10 + \sqrt{3})^{-1}$
 - $(3 + \sqrt{10})^{-1}$
 - $(3 + \sqrt{10})$
 - $(10 + \sqrt{3})$
3. $\lim_{x \rightarrow -1^+} \frac{\sin^{-1}(\sqrt{\pi} - \sqrt{\cos^{-1}x})}{\sqrt{1-x^2}}$ equals
- 0
 - 1
 - $\frac{1}{2\sqrt{\pi}}$
 - $\frac{1}{\sqrt{\pi}}$
4. If $f(x) = 3 + (1 + 7^{\frac{1}{1-x}})^{-1}$ then
- $\lim_{x \rightarrow 1^-} f(x) = 4$
 - $\lim_{x \rightarrow 1^+} f(x) = 3$
 - $\lim_{x \rightarrow 1^+} f(x) = 5$
 - f has irremovable discontinuity at $x = 1$
5. If $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$ then the value of $\lim_{x \rightarrow 1^+} \frac{f(1-h) - f(1)}{h^3 + 3h}$ is
- $\frac{-53}{3}$
 - $\frac{-22}{3}$
 - $\frac{53}{3}$
 - $\frac{22}{3}$
6. If the triangle formed by the lines $x^2 - y^2 = 0$ and the line $lx + 2y = 1$ is isosceles then $l =$
- 1
 - 2
 - 3
 - 0
7. Using L'Hospital's rule or otherwise evaluate the limit, $\lim_{x \rightarrow 0} \frac{(e^{2x^2} - 1 - 2x^2)(\cos x - 1)}{(\sin 3x - \ln(1 + 3x))x^4}$
8. Evaluate the limit $\lim_{x \rightarrow 0} \frac{e^x - \ln(x + e)}{e^x - 1}$ Use of L'Hospital's rule or surd expansion not allowed.
9. Find all real numbers t satisfying the equation $(3^t - 9)^3 + (9^t - 3)^3 = (9^t + 3^t - 12)^3$
10. Find $g'(3)$ if $g(x) = x \cdot 2^{h(x)}$ where $h(3) = -2$ and $h'(3) = 5$

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11. Find the value of expression $\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}$
12. Let $f(x) = a\cos(x+1) + b\cos(x+2) + \cos(x+3)$ where a,b,c are real . Given that f(x) has at least two zeros in the interval $(0, \pi)$ find all its real zeroes.
13. Calculate $\sin\left(\frac{1}{4}\arcsin\frac{\sqrt{63}}{8}\right)$
14. If θ is eliminated from the equation $a\cos\theta + b\sin\theta = c$ and $a\cos^2\theta + b\sin^2\theta = c$ show that the eliminant is $(a-b)^2(a-c)(b-c) + 4a^2b^2 = 0$
15. A triangle has side lengths 18,24 and 30 Find the area of the triangle whose vertices are the incentre circumcentre and centroid of the triangle.

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16. Find the real solutions to the system of equations
 $\log_{10}(2000xy) - \log_{10}x \cdot \log_{10}y = 4$
 $\log_{10}(2yz) - \log_{10}y \cdot \log_{10}z = 1$
 and $\log_{10}(zx) - \log_{10}z \cdot \log_{10}x = 0$
17. Prove that , $\cos^{-1}\left[1 - \frac{1 - \cos x}{12\cos x + 13}\right] = \pi - 2\cot^{-1}\left(\frac{1}{5}\tan\frac{x}{2}\right)$ where $x \in (0, \pi)$
18. Compute the value of $\cos\left(\frac{1}{4}\tan^{-1}\frac{24}{7}\right)$
19. If $g(x) = x^3 + px^2 + qx + r$ where p,q and r are integers. If $g(0)$ and $g(-1)$ are both odd, then prove that the equations $g(x) = 0$ cannot have three integral roots.
20. Sum the series $\cot^{-1}(2a^{-1} + a) + \cot^{-1}(2a^{-1} + 3a) + \cot^{-1}(2a^{-1} + 6a) + \cot^{-1}(2a^{-1} + 10a) + \dots$ to n terms Also find the sum of infinite terms. ($a > 0$)
21. Let $x = \frac{\sum_{n=1}^{44} \cos n^\circ}{\sum_{n=1}^{44} \sin n^\circ}$ find the greatest integer that does not exceed $100x$