

1. $\tan\theta = \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots \infty}}}$ where $\theta \in (0, 2\pi)$ find the possible value of θ [2]
2. Find the sum of the solutions of the equation $2e^{2x} - 5e^x + 4 = 0$ [2]
3. Suppose that x and y are positive numbers for which $\log_9 x = \log_{12} y = \log_{16}(x + y)$ If the value of $\frac{y}{x} - 2\cos\theta$, where $\theta \in (0, \frac{\pi}{2})$ find θ [2]
4. Using L'Hospitals or otherwise, evaluate the following limit $\lim_{x \rightarrow 0^+} \left(\frac{[1^2(\sin x)^x] + [2^2(\sin x)^x] + \dots + [n^2(\sin x)^x]}{n^3} \right)$ where $[.]$ denotes the greatest integer function.
5. Consider $f(x) = \frac{1}{\sqrt{b-a}} \frac{\sqrt{\frac{b-a}{a} \sin 2x}}{\sqrt{1 + (\sqrt{\frac{b-a}{a} \sin x})^2}} \sqrt{a + b \tan^2 x}$, for $b > a > 0$ and the functions $g(x)$ and $h(x)$ are defined, such that $g(x) = [f(x)] - \left\{ \frac{f(x)}{2} \right\}$ & $h(x) = \text{sgn}(f(x))$ for $x \in \text{domain of } f$, otherwise $g(x) = 0 = h(x)$ for $x \notin \text{domain of } f$, where $[x]$ is the greatest integer function of x and $\{x\}$ is the fractional part of x . Then discuss the continuity of g and h at $x = \frac{\pi}{2}$ and $x = 0$ respectively.
6. $\int \frac{x^2 \tan^{-1} x}{(1+x^2)^{5/2}} dx$
7. Using substitution only, evaluate $\int \text{cosec}^3 x dx$

M.M.:42

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Time : 55 Mints

1. If $\sin A = \frac{12}{13}$ Find the value of $\tan \frac{A}{2}$
2. The straight line $\frac{x}{a} + \frac{y}{b} = 1$ cuts the x axis and the y axis in A and B respectively and a straight line perpendicular to AB cuts them in P and O respectively. Find the locus of the point of intersection of AQ and BP.
3. If $\frac{\tan \theta}{\tan \theta - \tan 3\theta} = \frac{1}{3}$ find the value of $\frac{\cot \theta}{\cot \theta - \cot 3\theta}$
4. If a ΔABC is formed by the lines $2x+y-3=0$, $x-y+5=0$ and $3x-y+1=0$ then obtain a cubic equation whose roots are the tangent of the interior angles of the triangle.
5. Integrate $\int \frac{dx}{(a^2 - \tan^2 x)\sqrt{b^2 - \tan^2 x}}$ ($a > b$)
6. Let $\frac{d}{dx}(x^2y) = x - 1$ where $x \neq 0$ and $y = 0$ when $x = 1$. Find the set of values of x for which $\frac{dy}{dx}$ is positive.

M.M.:48

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Time : 70 Mints

1. Two circles of radii R and r are externally tangent. Find the radius of the third circle which is between them and touches those circles and their external common tangent in terms of R and r.
2. Let a matrix A be denoted as $A = \text{diag}(5^x, 5^{5^x}, 5^{5^{5^x}})$ then compute the value of the integral $\int (\det A) dx$
3. Evaluate $\int \frac{-x}{1+x} \frac{dx}{\sqrt{x+x^2+x^3}}$
4. If three distinct points, $(\frac{a^3}{a-1}, \frac{a^2-3}{a-1})$, $(\frac{b^3}{b-1}, \frac{b^2-3}{b-1})$, $(\frac{c^3}{c-1}, \frac{c^2-3}{c-1})$ are collinear then show that $abc + 3(a+b+c) = ab + bc + ca$
5. Integrate $\int \sqrt[3]{\tan x} dx$