

1. If  $\cos A, \cos B$  and  $\cos C$  are the roots of the cubic  $x^3 + ax^2 + bx + c = 0$  where  $A, B, C$  are the angles of a triangle then find the value of  $a^2 - 2b - 2c$  [5]
2. Find all functions,  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfying  $(xf(x) - 2F(x))(F(x) - x^2) = 0 \forall x \in \mathbb{R}$  where  $f(x) = F(x)$ . [5]
3.  $\int_{3/2}^2 \left(\frac{x-1}{3-x}\right)^{1/2} dx$  [5]
4. For  $a > 0, b > 0$  verify that  $\int_0^\infty \frac{\ln x}{ax^2 + bx + a} dx$  reduces to zero by a substitution  $x = 1/t$ . Using this or otherwise evaluate  $\int_0^\infty \frac{\ln x}{x^2 + 2x - 4} dx$  [7]
5.  $\int_0^\infty \left(\frac{\tan^{-1} x}{x}\right)^3 dx$  [8]