Multiple Choice Questions (MCQs)

Choose the correct option.

1. Which of the following relations is an example of a reflexive relation?

   (a) Less than
   (b) Greater than
   (c) Equal to
   (d) Not equal to

2. A relation R on the set $A = \{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3)\}$ is:

   (a) Reflexive only
   (b) Symmetric only
   (c) Transitive only
   (d) Reflexive, Symmetric, and Transitive

3. Which of the following relations is not symmetric?

   (a) Being a sibling of
   (b) Being a parent of
   (c) Being married to
   (d) Being a friend of

4. In a set of integers, the relation "is divisible by" is:

   (a) Reflexive and symmetric
   (b) Reflexive and transitive
   (c) Only transitive
   (d) Reflexive, symmetric, and transitive

5. Which of the following is an equivalence relation?

   (a) $<$ on $\mathbb{Z}$
   (b) $\leq$ on $\mathbb{Z}$
   (c) $=$ on $\mathbb{Z}$
   (d) $\neq$ on $\mathbb{Z}$

6. The relation R on the set of all humans by "has the same birthday as" is:

   (a) Not reflexive
   (b) Not symmetric
   (c) Not transitive
   (d) An equivalence relation

7. If a relation is both symmetric and transitive, then it is necessarily:

   (a) Reflexive
   (b) An equivalence relation
   (c) Not reflexive
   (d) None of the above

8. For any non-empty set A, the identity relation on A is:

   (a) Not symmetric
   (b) Not transitive
   (c) An equivalence relation
   (d) None of the above

Short Answer Questions (2 Marks)

9. Define a reflexive relation and give an example.

10. What does it mean for a relation to be symmetric? Provide an example.

11. Describe the transitive property of a relation with an example.

12. Explain what an equivalence relation is. Give an example.

13. Can a relation on a set be reflexive and not symmetric? Justify your answer.

Long Answer Questions (4 Marks)

14. Prove that the relation "is congruent to, modulo n" is an equivalence relation on the set of integers.

15. Given a set $A = \{1, 2, 3, 4\}$, define a relation R on A by $R = \{(a, b) \mid a$ is divisible by $b\}$. Determine if R is reflexive, symmetric, and transitive.

16. Show that if a relation R on a set A is reflexive, symmetric, and transitive, then R is an equivalence relation.

17. Consider the relation R on the set of all people in the world by "has the same mother as". Is R an equivalence relation? Justify your answer.

Answers

1. (c)

2. (d)

3. (b)

4. (b)

5. (c)

6. (d)

7. (b)

8. (c)

9. A reflexive relation on a set A is a relation where every element is related to itself. Example: $\leq$ on $\mathbb{Z}$.

10. A symmetric relation is such that if a is related to b, then b is related to a. Example: "is married to".

11. A relation is transitive if whenever a is related to b and b is related to c, then a is related to c. Example: $\leq$ on $\mathbb{R}$.

12. An equivalence relation is a relation that is reflexive, symmetric, and transitive. Example: Equality on any set.

13. Yes, a relation can be reflexive and not symmetric. For example, the "is less than or equal to" relation on $\mathbb{R}$.

14. *Proofs will vary.*

15. *Analysis will vary.*

16. *Proofs will vary.*

17. *Discussion will vary.*

Solution

Multiple Choice Questions (MCQs)

Choose the correct option.

1. Which of the following relations is an example of a reflexive relation?

   (a) Less than

   (b) Greater than

   (c) Equal to

   (d) Not equal to

   Answer: (c) Equal to

2. A relation R on the set $A = \{1, 2, 3\}$ given by $R = \{(1,1), (2,2), (3,3)\}$ is:

   (a) Reflexive only

   (b) Symmetric only

   (c) Transitive only

   (d) Reflexive, Symmetric, and Transitive

   Answer: (d) Reflexive, Symmetric, and Transitive

3. Which of the following relations is not symmetric?

   (a) Being a sibling of

   (b) Being a parent of

   (c) Being married to

   (d) Being a friend of

   Answer: (b) Being a parent of

4. In a set of integers, the relation "is divisible by" is:

   (a) Reflexive and symmetric

   (b) Reflexive and transitive

   (c) Only transitive

   (d) Reflexive, symmetric, and transitive

   Answer: (b) Reflexive and transitive

5. Which of the following is an equivalence relation?

   (a) $<$ on $\mathbb{Z}$

   (b) $\leq$ on $\mathbb{Z}$

   (c) $=$ on $\mathbb{Z}$

   (d) $\neq$ on $\mathbb{Z}$

   Answer: (c) $=$ on $\mathbb{Z}$

6. The relation R on the set of all humans by "has the same birthday as" is:

   (a) Not reflexive

   (b) Not symmetric

   (c) Not transitive

   (d) An equivalence relation

   Answer: (d) An equivalence relation

7. If a relation is both symmetric and transitive, then it is necessarily:

   (a) Reflexive

   (b) An equivalence relation

   (c) Not reflexive

   (d) None of the above

   Answer: (b) An equivalence relation

8. For any non-empty set A, the identity relation on A is:

   (a) Not symmetric

   (b) Not transitive

   (c) An equivalence relation

   (d) None of the above

   Answer: (c) An equivalence relation

Short Answer Questions (2 Marks)

9. Define a reflexive relation and give an example.
   Answer: A reflexive relation on a set A is a relation where every element is related to itself. Example: $\leq$ on $\mathbb{Z}$.

10. What does it mean for a relation to be symmetric? Provide an example.
   Answer: A symmetric relation is such that if a is related to b, then b is related to a. Example: "is married to".

11. Describe the transitive property of a relation with an example.
   Answer: A relation is transitive if whenever a is related to b and b is related to c, then a is related to c. Example: $\leq$ on $\mathbb{R}$.

12. Explain what an equivalence relation is. Give an example.
   Answer: An equivalence relation is a relation that is reflexive, symmetric, and transitive. Example: Equality on any set.

13. Can a relation on a set be reflexive and not symmetric? Justify your answer.
   Answer: Yes, a relation can be reflexive and not symmetric. For example, the "is less than or equal to" relation on $\mathbb{R}$.

Long Answer Questions (4 Marks)

14. Prove that the relation "is congruent to, modulo n" is an equivalence relation on the set of integers.
   Answer: The relation "is congruent to, modulo n" is an equivalence relation on the set of integers because it is reflexive (every integer is congruent to itself modulo n), symmetric (if a is congruent to b modulo n, then b is congruent to a modulo n), and transitive (if a is congruent to b modulo n, and b is congruent to c modulo n, then a is congruent to c modulo n).

15. Given a set $A = \{1,2,3,4\}$, define a relation R on A by $R = \{(a,b) \mid a$ is divisible by $b\}$. Determine if R is reflexive, symmetric, and transitive.
   Answer: The relation R on the set $A = \{1,2,3,4\}$ defined by $R = \{(a,b) \mid a$ is divisible by $b\}$ is reflexive (every element is divisible by itself), not symmetric (if 2 is divisible by 1, it doesn't imply 1 is divisible by 2), and transitive (if 4 is divisible by 2 and 2 is divisible by 1, then 4 is divisible by 1).

16. Show that if a relation R on a set A is reflexive, symmetric, and transitive, then R is an equivalence relation.
   Answer: A relation R on a set A is an equivalence relation if it is reflexive, symmetric, and transitive. This is a fundamental property that does not require proof beyond the definitions of these three properties.

17. Consider the relation R on the set of all people in the world by "has the same mother as". Is R an equivalence relation? Justify your answer.
   Answer: The relation "has the same mother as" on the set of all people in the world is not necessarily an equivalence relation because it is not necessarily reflexive (an individual may not be considered to have the same mother as themselves if the definition requires at least two distinct individuals), it is symmetric (if person A has the same mother as person B, then person B has the same mother as person A), and it is transitive (if person A has the same mother as person B, and person B has the same mother as person C, then person A has the same mother as person C). However, considering reflexivity in a broader sense that includes an individual in relation to themselves, it could be seen as an equivalence relation. The confusion here might arise from how we interpret the reflexivity in this context.